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Scientific Conference

"Research in Mathematics Education"

Proceedings

Edited by Jasmina Milinković and Zoran Kadelburg

Mathematical Society of Serbia

Belgrade, Serbia

May 10 – 11, 2019

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Published by: Mathematical Society of Serbia, 35/IV, Kneza Mihaila, Belgrade, Serbia

www.dms.rs

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Since all papers are written in English which is, for most authors, not their first language, the responsibility for spelling and grammar lies with the authors of the papers themselves.

ISBN 978-86-6447-017-9

CIP - Каталогизација у публикацији
Народна библиотека Србије, Београд

371.3::51(082)

SCIENTIFIC Conference "Research in Mathematics Education" (2019 ; Beograd)
Proceedings / Scientific Conference "Research in Mathematics Education", Belgrade, Serbia May 10 - 11, 2019 ; edited by Jasmina Milinković and Zoran Kadelburg. - Belgrade : Mathematical Society of Serbia, 2019 (Valjevo : Topalović). - 141 str. : ilustr. ; 30 cm

Na vrhu nasl. str.: European Society for Research in Mathematics Education. - Tiraž 100. - Bibliografija uz svaki rad.

ISBN 978-86-6447-017-9

a) Математика -- Настава -- Методика -- Зборници

COBISS.SR-ID 281861388

Circulation: 100

Printing: „Štamparija Topalović“ Valjevo

Preview

These proceedings represent a selection of papers presented at the scientific conference held in Belgrade, Serbia, May 10 – 11, 2019. The conference was organized upon the initiative of ERME to support the development of the research community in Eastern Europe. In an effort to follow the international trend of intensification of research in the field of mathematical education, the Mathematical Society of Serbia took the initiative to organize a scientific conference devoted to this issue. The major aim was to introduce and exchange ideas and results of research in mathematics education among math educators in Serbia and neighbouring countries. The themes of plenary presentations traced directions of the research and presented selected findings and perspectives on the proposed topic fields: 1) place and role of different participants in the research in mathematics education, 2) how to use lessons from history and research in educational practice, 3) research methods in mathematics education. There were 41 participants, 3 plenary lectures and 15 reports. The full program is included in these proceedings (p. 134). The proceedings contain two plenary lectures and 9 papers from the authors who decided to send contributions for the proceedings and passed the peer-reviewing process.

We are grateful to ERME and Ministry of Education, Science and Technological Development of Serbia for offering financial support, plenary speakers, contributors, and all participants for taking an active part in the conference, and colleagues who took part in the reviewing process.

Editors

Belgrade, December 2019

Content

The application of modern technology in teaching and learning stereometry

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The paper analyzes the possibilities of applying a three-dimensional view of geometric objects within the dynamic software GeoGebra, during the teaching and learning stereometry. The research included students in the final (eighth) grade of elementary school. The GeoGebra software was applied in introducing students in the formation and the elements of a right circular cylinder. The method of collaborative learning was applied. The students worked in small groups and they had the possibility to independently create dynamic worksheets, by using GeoGebra software, which they used for observing a three-dimensional view of solid figures (cylinder and prism), analyzing and comparing their elementary properties. After finishing group work, the students presented their results, after which they had the opportunity to discuss their observations. The results of students' work were analyzed. Guidelines for further research as well as for the implementation of the described approach in teaching are given.

Keywords: Cylinder; Dynamic software; Stereometry; Three-dimensional view.

Introduction

Solid geometry (stereometry) is a very important mathematics topic in the most of the curricula. Its studying usually starts during elementary education and it is also present in secondary and tertiary education. Within this topic, the students are being introduced to the solid figures (cube, cuboid, prism, pyramid, cylinder, cone, sphere...) and their properties.

In Serbian educational system, stereometry contents are being studied during primary education, when the most of students are about 10 years old. At this level, students become familiar with the cube and cuboid, their surface areas and volumes. Also, the students are being introduced to the concepts of cylinder, cone and sphere. It should be mentioned that children are being introduced to the solid figures and their names earlier, even during preschool education. Later, at final grade of elementary school (when the most of the students are 14 years old), solid figures and their properties are being studied more in detail. At that level, the students are being introduced to right figures (actually, they learn what oblique figures are, but not in detail). Finally, in the third grade of high school, they are being introduced to solid figures and their properties much in detail.

The teachers and the students usually encounter difficulties in learning solid geometry. The students have problems with understanding the contents and the teachers have problems to explain them different concepts and their properties in this area. Most of these difficulties are related to the spatial reasoning. Usually, students have problems to identify all necessary elements of the observed solid figures, to understand their generation, to perceive a net of individual figure. Especially, the students have difficulties to perceive an intersection of the figure and plane. Earlier research and experience showed that some of these difficulties are usual for many students, even for those who master the material at the advanced level of achievements. It can be said that the students have problem to imagine the figures they don't see.

Usually, the teachers try to find different ways to overcome these difficulties. One of the most important things is visualization. The teachers have always been applied different methods to visualize solid figures. The sketch is usual, but not so effective, because it is not clear enough when we show 3D object in two dimensions. Better results have been achieved when three-dimensional models or wear-frame models were used. Especially wear-frame models, because their application enables different elements, such as diagonals and intersections, to be shown. However, the application of modern technology creates the new possibilities in visualization.

In this paper, some possibilities of using modern technology in teaching and learning stereometry are presented, and the influence of its application on students' understanding of a cylinder and its properties is analyzed.

Theoretical background

The understanding of the solid figures and their properties depends on the spatial reasoning and spatial ability of the individual. Students' spatial abilities, consisted of spatial visualization, orientation and relations factors, are a strong predictive factor of the reasoning in solid geometry thinking (Pittalis & Christou, 2010). Clements and Battista (1992) claim that "spatial reasoning consists of the set of cognitive processes by which mental representations of spatial objects, relationships, and transformations are constructed and manipulated".

Earlier studies have shown that spatial reasoning intervention improves the students' performance in geometry, especially solid geometry (Pittalis & Christou, 2010; Ramful, Lowrie, & Logan, 2017). It is shown that improvement in children's spatial ability has a positive influence to their mathematics ability in general (Cheng & Mix, 2014).

However, numerous difficulties are usually encountered in teaching and learning stereometry contents. Many students have difficulties to understand and describe the properties of 3D objects, draw the correct two-dimensional representation and the net of the observed object (Marchis, 2012). Also, many students, especially those who mater the material on the elementary or intermediate level of achievement, have problems to understand the generation of the solid figures and to perceive an intersection of the figure and plane. This is also difficult topic for teachers, because they have to help their students in overcoming encountered difficulties (Salman, 2009).

Most of the teachers try to find a way to explain difficult teaching contents and to provide a suitable learning environment to their students. Usually, the teaching of solid geometry is being enhanced by improving the multiple representations of the observed objects. When it comes to the stereometry, the most importance is given to the representations which enable visualization of the solid figures because it is considered that visualization has a significant importance in describing and explaining the properties of solid figures (Kurtuluş, & Uygan, 2010). Particular importance is given to the realistic representations, such as 3D models of the solid figures and, especially in recent times, to the manipulative representations (Nakahara, 2008).

Multiple representations in teaching mathematics have gained significance with the development of technology. Improving the teaching of mathematics and, especially, stereometry, by using appropriate software, was the topic of many researches (Baki, Kosa, & Guven, 2011; Hwang & Hu, 2013; Stols, 2012). Most of the software packages, which are appropriate for using in teaching and learning stereometry, enable multiple representations of solid figures, with the accent on graphical representation. Visualization of solid figures is being further enhanced by the possibility of rotating the observed figure. Also, dynamic geometry software enables creating the manipulative representation of the observed object. Manipulative representation provides a possibility that measures of some elements of the observed solid figure be defined by slider, which moving causes the changes in the measure of depended element. As a result, some properties of the observed figure are being changed. The use of manipulative representation enables analysis of the influence of individual elements to the properties of the observed solid figure.

The application of the physical manipulatives and dynamic geometry software in teaching is more effective in developing the students' spatial visualization skills than traditional instruction. Also, instructions based on dynamic geometry software proved to be more effective than the instructions based on the physical manipulatives (Baki, Kosa, & Guven, 2011). The use of dynamic software, adjusted for dealing with solid figures, stimulates geometric cognitive growth of the pre-service teachers. Also, it can contribute to better understanding the properties of solid figures, especially when it comes to students who master the material on elementary and intermediate, or upper-intermediate level of achievements (Stols, 2012).

The use of software in a collaborative environment contributes to the students' ability in geometric problem solving, especially when it comes to the 3D objects. Namely, software which provides multiple representations of the observed object contributes enable properties of that object to be observed from different points of view and peer learning, which is present in a collaborative learning environment, is useful to facilitate geometric problem solving by sharing ideas and exploring multiple representations (Hwang & Hu, 2013).

One of the dynamic software packages, which are appropriate for creating the multiple representations of the solid figures, is *GeoGebra*. It enables 3D view, including manipulative representation of the object created in this view. This educational software proved to be successful in teaching geometry, but also other branches of mathematics. The use of *GeoGebra* software for visualization and working within multiple representations environment was the topic of the numerous studies (Arzarello, Ferrara, & Robutti, 2012; Božić, Takači, & Stankov, 2019; Lazarov, 2012; Tran, Nguyen, Bui, & Phan, 2014), which proved its benefits.

The application of *GeoGebra* software contributes to the successful implementation of problem-based and discovery learning. It helps the teachers in realization the teaching process, but also helps the students during individual learning and improves the interaction between teacher and students (Tran et al., 2014). Dynamic representations in *GeoGebra* environment can be successfully applied in mathematical modeling in teaching, i.e. in problem solving activities, for creating approximations of the models (Arzarello, Ferrara, & Robutti, 2012). It is proved that the application of *GeoGebra* and the use of dynamic multiple representations in *GeoGebra* environment significantly contributes to the students' better achievements in examining functions with parameters (Božić, Takači, & Stankov, 2019).

Besides *GeoGebra*, there are some other software packages appropriate for the use in teaching stereometry. Opposite most of them, *GeoGebra* is free software, so the students and the teachers don't have difficulties with its availability. Also, *GeoGebra* is easy to use and the students can be easily trained for its application (Hohenwarter, Jarvis, & Lavicza, 2009).

Research question

Previous research showed that the application of dynamic geometry software with possibility of 3D view can improve students' spatial ability (Baki, Kosa, & Guven, 2011; Stols, 2012). Also, it is proved that work in a collaborative environment has a positive influence on students' achievements in learning geometry (Hwang & Hu, 2013). Because of all mentioned, it is decided to examine the influence of the application of dynamic software *GeoGebra* within a collaborative learning environment on students' understanding of the cylinder concept and its properties.

The main question of this research is:

Does the application of dynamic geometry software with 3D view, within collaborative learning environment, contribute to students' better understanding the cylinder concept, its generation, elements and properties?

Methodology

This research presents a case study. The research is conducted with 24 students of the final (8^{th}) grade of the Elementary school "Vuk Karadžić" in Novi Sad, Serbia, in the 2017/2018 school year. At first, the students were divided into four-member collaborative groups. Eight collaborative groups were formed at the beginning of the class, taking into account students' previous achievements so in each group there were students who reached elementary, intermediate and advanced level of achievements and the heterogeneity of the groups has been achieved. They worked in the digital classroom and every group used one computer.

The study covered an introduction with the cylinder concept and its properties and students worked two school hours (90 minutes). A cylinder concept and its properties were chosen as a teaching topic to be processed within this experiment. *GeoGebra* software package was chosen for creating the dynamic multiple representations of the observed solid figure. Qualitative analysis of the students' work is carried out. The students work within *GeoGebra* environment was observed by the teacher. Cooperation and discussion among the students in every collaborative group, but also discussion among the groups and with teacher was the subject of observation. The dynamic worksheets, created by the students within *GeoGebra* environment, were saved and analyzed by the teacher.

The analysis of the students' work

After forming the collaborative groups, the students were introduced with the topic. The concept of a cylinder is not unknown to students of the final grade of elementary school (they are able to recognize solid figures during primary education). So, at first, previously learned was recalled and students discussed about cylinder and its properties with the teacher and with each other. Most of the students tried to define a cylinder by explaining its properties. After that, they were introduced in the cylinder concept in detail; with the accent on the right cylinder (oblique cylinder was only mentioned – it is being studied in detail in high school). In this part of the lesson, the dialog method was dominated. But, this method was present during all 90 minutes, less or more.

After the introduction, the teacher provided necessary instructions for students. They are informed about what they are expected to do and how to use *GeoGebra* software. The eighth grade students are usually familiar with *GeoGebra*, because they learn about this software within computer science. The teacher gave the instructions about the use of 3D view in *GeoGebra* and suggested the order by which students should analyze cylinder and its elements.

In the most of the groups, the students organized their work in such way that one of them worked within *GeoGebra* dynamic worksheets, the other gave the instructions and the rest two noted the results. During the work process, the students usually changed their roles within the groups, so every student had the opportunity to work with *GeoGebra* software. In all the groups, at the beginning, the student with higher achievements was in charge for giving instructions, but later, during the learning process, the other students were giving their suggestions and they made the conclusions together, by discussion.

In the next phase, the students discussed about the generation of a cylinder. The teacher had to show them generation of a cylindrical surface, but some of the students knew that cylinder can be generated by rotation of a rectangle. They explained that a cylinder can be generated by "rotation of a rectangle around its side." After the teacher's instructions, the students used *GeoGebra* software in order to create dynamic worksheets which show generation of a cylinder by rotation of a rectangle. When they had some difficulties, they could get help of the teacher. One example of these worksheets, created by the students, is shown in Figure 1. After they used *GeoGebra* dynamic worksheets to observe generation of a cylinder, some students noticed that the cylinder also can be generated by rotation of a rectangle around its symmetry axes, but also around any line which is parallel to its side.

After that, the students observed different intersections of a cylinder and a plane and discussed about the figures obtained. In this part, the students also used *GeoGebra* software performances in the best possible way. They used the appropriate options of *GeoGebra* 3D view and created the representations of the requested intersections. They created different intersections and discussed the about which figures could be the intersections of a cylinder and a plane. Most of the students observed the cases when the intersection is a rectangle or a circle, especially the case when the observed plane contains the axis of the cylinder. One example of such students' work is shown in Figure 3.

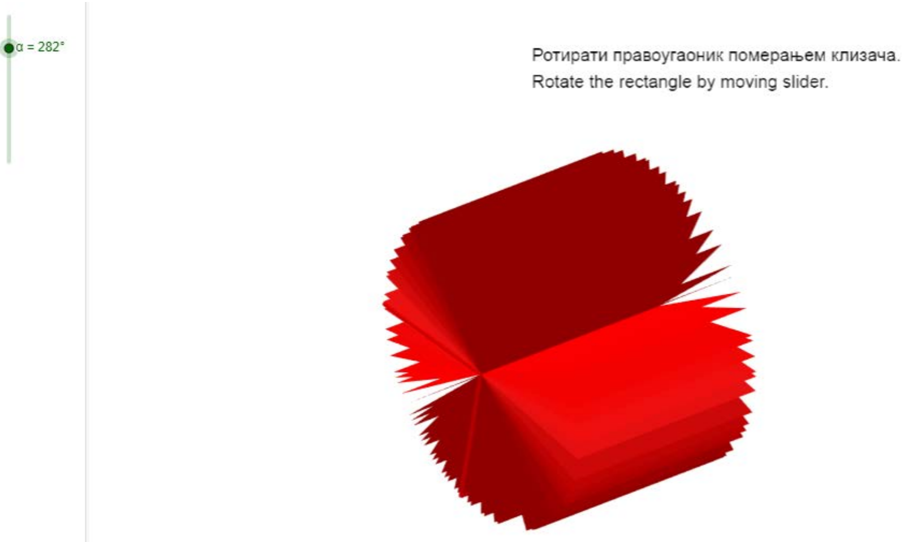

Figure 1: Students' work – rotation of a rectangle

They concluded that there are infinitely many possibilities of the intersection with a plane, but, if the plane contains an axis of a cylinder, all those intersections (rectangles) are congruent and in the other cases they are non-congruent. The students who observed the intersections of a cylinder with a plane parallel to the planes of the bases concluded that those intersections are the circles. Three groups of students observed the other cases and concluded that, in those cases, intersections of a cylinder with a plane can be some "irregular" figures (they used term irregular because those figures were unknown to them). Only one student recognized the ellipse as the intersection.

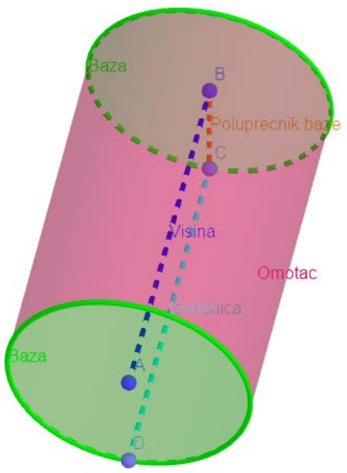

Figure 2: Students' work – elements of a cylinder

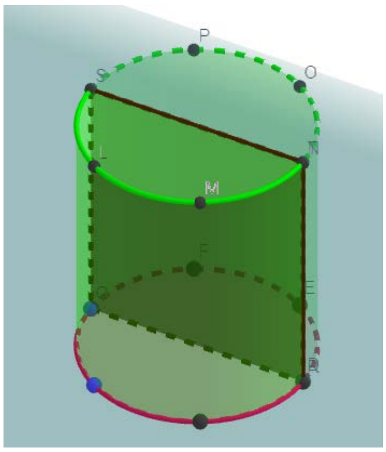

Figure 3: Students' work – intersection with a plane

The next task for the students was to analyze a net of a cylinder. Some of the students had an idea how a net looks like, but they couldn't create it within *GeoGebra* software. This was expected, because it is too complicated for them. Namely, for creating a net of a cylinder, knowledge of elementary school students is not enough. Because of that, the teacher showed them a net of a cylinder, created within *GeoGebra* environment and they discussed about it. As a result of this discussion, the students concluded how to determine the surface area of a cylinder. They explained that the surface area of a cylinder presents a sum of the surface areas of its bases (two circles) and the surface area of its lateral area (rectangle, if a cylinder is "opened").

By following teachers' instructions, the students created new worksheets, which are used for the comparison of a cylinder and a prism. The students were familiar with the prism and its properties, and now they compared it with the cylinder properties. In Figure 4, an example of comparison between cylinder and prism is shown. Many students noticed that "when the number of prism's sides is being increased, a prism looks more and more like a cylinder". One student said that "cylinder can be considered as a prism with the infinitely many sides." As a result, they concluded how to determine the volume of a cylinder, i.e. they explained that the volume of a cylinder can be determined in the same way as the volume of a prism.

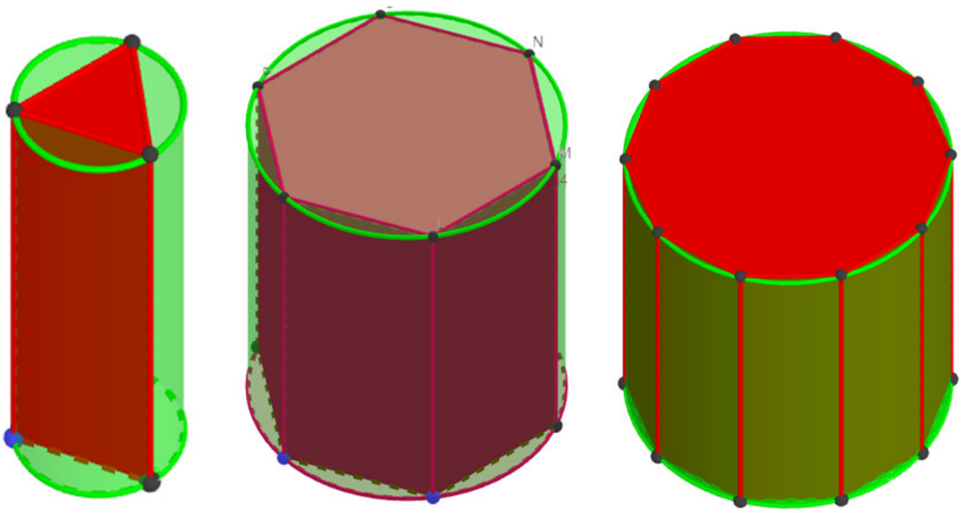

Figure 4: Students' work – cylinder and prism

During the learning process, there was intense discussion among the students and, also, they asked a number of questions to the teacher. All students actively participated in the learning process and discussion, which is not the case when a classical teaching method is applied. Usually, when the classical teaching method is applied, only several students participate in discussion and the others passively follow the lesson. Also, there were no students who were not motivated to follow the lesson. The students also were interested in *GeoGebra* 3D view and its possibilities and the teacher informed them about appropriate information source and the literature. The analysis of their *GeoGebra* dynamic worksheets showed that they have an excellent deal with this software package.

Conclusions

During two classes which were covered by the research, the students work was carefully monitored by the teacher. Special attention is paid to the students' work within collaborative groups, the use of *GeoGebra* software, but also their mutual cooperation, discussion and exchanging ideas and the organization of the group work, in general. Besides that, discussion and cooperation between different groups were also observed and analyzed.

At the beginning of their work within *GeoGebra* environment, the students split their roles it the way described above. Later, during the work, they changed their roles and adjusted them to the current needs. All students, in all the groups, were included in the group work and all of them performed their duties appropriately. It is important to mention that the students with lower achievements, at the beginning, were unsure of themselves, but later they also participated in the group work, not less than the other group members. These students usually worked within *GeoGebra* dynamic worksheets, following instructions from the other students. During the work, they were discussing in order to reach the correct conclusions.

At the beginning, when they some difficulties, especially when it comes to the use of *GeoGebra* software, the students of some group asked the teacher for help. Later, they usually asked the members of the other groups. In this way, cooperation between different groups has been achieved. Also, during the discussion about the results of their work, the students of different groups expressed their opinions and disagreements with the conclusions of the other students.

As regards their conclusions about the properties of a cylinder, its generation, elements and intersections, the students reached all necessary conclusions, more independently and with significantly less help of the teacher than it is the case when classical teaching methods are applied. Besides, some of the students came to the ideas and reached the conclusions which are usually not expected from the students of their age.

From the above, it can be concluded that the use of software helped students to visualize the observed solid figure and to perceive its properties from different points of view, by using dynamic multiple representations. Also, the collaborative learning method, applied in the observed learning process, helped students to correct some mistakes and to, through the exchange of their ideas, reach the correct conclusions. Finally, it can be concluded that the application of dynamic geometry software with 3D view, within a collaborative learning environment, contributes to students' better understanding the cylinder concept, its generation, elements and properties, which presents positive answer to the research question. Dynamic geometry software, which enables 3D view, should be applied in teaching solid geometry contents, when there are conditions for its application and when a teacher estimates that it could be helpful. Also, collaborative learning method can be useful in teaching and learning these contents, because it enables discussion and exchanging the ideas among the students, which encourage active participation of every student in the learning process.

Some future research should be conducted with a larger sample, with the experimental and the control group. The participants of the research should be tested and their test results should be quantitatively analyzed. The data obtained in that way would confirm if the influence of the application of dynamic geometry software causes statistically significant advantage in the achievements of the students who used the computer in comparison with the students who learned in a classical way. Also, the high school students should be covered by the future research, because of the difficulties occurred in teaching and learning stereometry contents in the high school, but also it should be examined the influence of modern technology application on the achievements of the younger students, at the beginning of their introduction with the stereometry concepts.

References

Arzarello, F., Ferrara, F., & Robutti, O. (2012). Mathematical modelling with technology: the role of dynamic representations. *Teaching Mathematics and Its Applications*, 31(1), 20-30.

Baki, A., Kösa, T., & Güven, B. (2011). A comparative study of the effects of dynamic geometry software and physical manipulatives on pre-service mathematics teachers' spatial visualization skills. *British Journal of Educational Technology*, 42(2), 291–310.

Božić, R., Takači, Đ., & Stankov, G. (2019). Influence of dynamic software environment on students' achievement of learning functions with parameters. *Interactive Learning Environments*, DOI: 10.1080/10494820.2019.1602842

Cheng, Y. L. & Mix, K. S. (2014). Spatial Training Improves Children's Mathematics Ability, *Journal of Cognition and Development*, 15(1), 2-11.

Clements, D. H., & Battista, M. T. (1992). Geometry and spatial reasoning. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 420-464). New York: Macmillan.

Hohenwarter, M., Jarvis, D., & Lavicza, Z. (2009). Linking Geometry, Algebra, and Mathematics Teachers: GeoGebra Software and the Establishment of the International GeoGebra Institute, *International Journal for Technology in Mathematics Education*, 16(2), 83-86.

Hwang, W. Y., & Hu, S. S. (2013). Analysis of peer learning behaviors using multiple representations in virtual reality and their impacts on geometry problem solving. *Computers and Education*, Vol. 62, 308-319.

Kurtuluş, A., & Uygan, C. (2010). The effects of Google Sketchup based geometry activities and projects on spatial visualization ability of student mathematics teachers. *Procedia-Social and Behavioral Sciences*, 9, 384-389.

Lazarov, B. (2012). An Approach to Incorporate Dynamic Geometry Systems in Secondary School —Model with Module. *The Teaching of Mathematics*, 15(1), 21-31.

Marchis, I. (2012). Preservice primary school teachers' elementary geometry knowledge. *Acta Didactica Naporensia*, 5(2), 33-40.

Nakahara, T. (2008). "*Cultivating mathematical thinking through representation-utilizing the representational system*", APEC-TSUKUBA International Congress, Japan.

Pittalis, M., & Christou, C. (2010). Types of reasoning in 3D geometry thinking and their relation with spatial ability. *Educational Studies in Mathematics*, 75(2), 191-212.

Ramful, A., Lowrie, T., & Logan, T. (2017). Measurement of Spatial Ability: Construction and Validation of the Spatial Reasoning Instrument for Middle School Students. *Journal of Psychoeducational Assessment*, *TBC*, 1-19.

Salman, M. F. (2009). Active Learning Techniques (ALT) in a Mathematics Workshop; Nigeria Primary School Teacher's Assesement. *International Electronic Journal of Mathematics Education*, 4(1), 23-35.

Stols, G. (2012). Does the use of technology make a difference in the geometric cognitive growth of pre-service mathematics teachers? *Australasian Journal of Educational technology*, 28(7), 1233-1247.

Tran, T., Nguyen, N. G., Bui, M. D., & Phan, A. H. (2014). Discovery Learning with the Help of the GeoGebra Dynamic Geometry Software. *International Journal of Learning, Teaching and Educational Research*, 7(1), 44-57.

Photo memories

Invited speakers

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Department of Design Engineering & Mathematics, Faculty of Science and Technology, Middlesex University, London, United Kingdom

Dr Patrick Barmby

No More Marking

Left Zoran Kadelburg, Jasmina Milinković and Vojislav Andrić

Right Plenary Panel

Below Auditorium

Auditourium

Coffee break

On Kalemegdan terrace

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